Pi Challenge

March 2023

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– This challenge is open for all students at university level.

– The solutions should be sent no later than **09.03.2023** by e-mail to mcm@nhsm.edu.dz.

– All solutions must be correct, complete and well written.

– The subject of your email should be written as follows

"FirstName/FamilyName/Pi-challenge".

– The email must contains all the following information: **Full name**, **Affiliation** (higher school, institute, university, ...), **Study level** and **Phone number**.

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Solve all the problems below.

1. Let $f \in \mathcal{C}^2(\mathbb{R})$ such that $f(x), f'(x)$ and $f''(x)$ are all strictly positive for every $x \in \mathbb{R}$. Show that if $f''(x) \leq f(x)$ for every $x \in \mathbb{R}$, then $f'(x) < \sqrt{2}f(x)$ for every $x \in \mathbb{R}$.

2. Find, with proof, the digit in the position $n+1$ after the decimal point in $\sqrt{N}$, where N is the positive integer with $2n$ digits that are all ones, i.e.,

$$N = \underbrace{111\cdots111}_{2n}, \quad n \geq 1.$$

3. Find the supremum and the infimum of

$$|1+z| + \left|1-z+z^2\right|,$$

where $z \in \mathbb{C}$ and $|z| = 1$.